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Note on central extensions and Leopoldt's conjecture

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Introduction

Let k be an algebraic number field of finite degree, and ℓ be a prime number. Throughout this paper, we always assume

$$(*) \qquad \sqrt{-1} \in k \quad \text{when} \quad \ell = 2.$$

We denote by $G(K/k)$ the Galois group of a Galois extension K/k. Denote by $k^{(\ell)}$ the maximal ℓ-extension of k unramified outside ℓ. Then it is well-known that Leopoldt's conjecture for k and ℓ is equivalent to $H^{-3}(G(k^{(\ell)}/k),\mathbb{Z}) = 0$. This is connected with a certain problem of central extensions through the relationship between the structure of the Galois group of a central extension and (the dual of) Schur's multiplicator $H^{-3}(G,\mathbb{Z})$ (Theorem 4). The problem is reduced in Section 3 to a simpler case (Theorem 8).

§1. Leopoldt's conjecture and abundant central extensions.

For any pro-finite group G and a natural number n, the cohomology group $H^{-n}(G,\mathbb{Z})$ of minus dimension is defined by

$$H^{-n}(G,\mathbb{Z}) \simeq \varprojlim H^{-n}(G/U_\lambda,\mathbb{Z}),$$

where U_λ runs over open subgroups of G of finite index, and $\varprojlim$ is of the deflation map. Then we have $H^{-n}(G,\mathbb{Z}) \simeq H^n(G,\mathbb{Z})^\wedge$. Hence $H^{-3}(G,\mathbb{Z}) \simeq H^3(G,\mathbb{Z})^\wedge \simeq H^2(G,\mathbb{Q}/\mathbb{Z})^\wedge$, which is called (the dual of) Schur's multiplicator of G.

For a tower of Galois extensions $M \supset K \supset k$, denote by $K^*_{M/k}$ the genus field of K/k in M, which is by definition, the composite of K and the maximal abelian extension of k in M. Denote by $\hat{K}_{M/k}$ the maximal central extension of K/k in M, namely the maximal extension in M whose Galois group over K is contained in the center of the Gaois group over k. Then we have the following theorem (Cf. Heider [3, §2], Furuta [1,Theorem 5]).

THEOREM 1.

$$G(\hat{K}_{M/k}/K^*_{M/k}) = \frac{H^{-3}(G(K/k),\mathbb{Z})}{\mathrm{Def}_{G(M/k)\to G(K/k)}\ H^{-3}(G(M/k),\mathbb{Z})} .$$

We call M *abundant for* K/k when $G(\hat{K}_{M/k}/K^*_{M/k}) \simeq H^{-3}(G(K/k),\mathbb{Z})$, namely $\mathrm{Def}_{G(M/k)\to G(K/k)}\ H^{-3}(G(M/k),\mathbb{Z}) = 0$.

For a Galois extension M/k, it follows from Theorem 1 and the definition of cohomology groups of pro-finite groups that $H^{-3}(G(M/k),\mathbb{Z}) = 0$ if and only if M is abundant for any finite Galois extension K over k contained in M.

Now denote by $\bar{k}$ the algebraic closure of k. Then it is well-known that

$$(1.1) \qquad H^{-3}(G(\bar{k}/k),\mathbb{Z}) = 0.$$

(Cf. Serre [9, Theorem 4], Heider [3, §5], Yamashita [10, Theorem 3], Miyake [8]). Hence $\bar{k}$ is abundant for any Galois extension K/k, and we have

THEOREM 2. $G(\hat{K}_{\bar{k}/k}/K^{*}_{\bar{k}/k}) \simeq H^{-3}(G(K/k),\mathbb{Z})$.

Now we are interested how small the abundant extension for K/k, whose existence is guaranteed as above, can be chosen, and especially in the following problem:

PROBLEM *For any Galois extension* K/k, *does there exist an abundant extension* M *for* K/k *such that only prime divisors ramified in* K/k *are ramified in* M/K ?

The above problem is closely related to Leopoldt's conjecture e.g. as follows.

We assume always (*) as in Introduction, and denote by $k^{(\ell)}$ the maximal ℓ-extension unramified outside ℓ. Then the following theorem is well-known (Cf. Heider [4, Satz 6, Bemerkung], Heider [5, Satz 11], Iwasawa [6], Kuz'min [7, Theorem 7.2]).

THEOREM 3. *Under the assumption* (*), *Leopoldt's conjecture for* ℓ *is true for* k *if and only if*

$$H^{-3}(G(k^{(\ell)}/k),\mathbb{Z}) = 0.$$

By the remark after Theorem 1, we have

THEOREM 4. *Under the assumption* (*), *Leopoldt's conjecture* for ℓ *is true for* k *if and only if* $k^{(\ell)}$ *is abundant for any Galois extension of finite degree over* k *contained in* $k^{(\ell)}$.

§2. Central extensions for a sequence of fields.

Let K/k be an ℓ-extension of finite degree. Then there is a sequence of extensions $k = K_0 \subset K_1 \subset \cdots \subset K_t = K$ such that K_{i+1}/K_i is cyclic of degree ℓ and each K_i is normal over k. Denote by $\hat{K}_i$ the maximal central extension of K_i/k in M. At first we reduce the structure of $G(\hat{K}_{M/k}/K^{*}_{M/k})$ to that of $G(\hat{K}_{i+1}/\hat{K}_i)$.

Let $M \supset L \supset K \supset k$ be a tower of Galois extensions over

k, and assume that M is abelian over K. Put $G = G(L/k)$, $H = G(K/k)$, $A = G(M/K)$ and $B = G(M/L)$. Then A and B are H-module and G-module respectively by means of conjugation. Let I_G and I_H be the augmentation ideals of the group rings $\mathbb{Z}[G]$ and $\mathbb{Z}[H]$ respectively. Denote by $L^{(i)}$ and $K^{(i)}$ be the extensions of L and K in M corresponding to $I_G^i B$ and $I_H^i A$ respectively. Note that $L^{(1)} = \hat{L}_{M/k}$ and $K^{(1)} = \hat{K}_{M/k}$.

Let $H_0 = H/[H,H]$, where $[H,H]$ is the commutator subgroup of H. For $\tau \in H$ and $a \in A$, denote by $\bar{\tau}$ and $\bar{a}$ the class of H_0 and A/B which contain τ and a respectively. Set

$$R(H,A,B) = \langle \prod_\tau (\bar{\tau} \otimes \bar{a}_\tau) \in H_0 \otimes (A/B)\ ; \prod_\tau a_\tau^{\tau - 1} = 1 \rangle$$

where $\otimes$ stands for the tensor product over $\mathbb{Z}$ by means of the exponential map.

Theorem 5. *Notation being as above, we assume that* $G(L/K)$ *is contained in the center of* $G(K/k)$. *Then we have*

(1) $$K^{(i+1)} \supset L^{(i)} \supset K^{(i)}$$

(2) $$G(\hat{L}_{M/k}/\hat{K}_{M/k}) \simeq \frac{H_0 \otimes (A/B)}{R(H,A,B)}$$

Proof. (1) Put $G_1 = G(L/K)$. Then $H = G/G_1$. Since A is abelian, we have $b^{g_1} = b$ for $b \in B$ and $g_1 \in G_1$. Hence we can treate B as H-module, and we have $I_G^i B = I_H^i B$. Then

$I_H^iA \supset I_H^iB = I_G^iB$. Hence $K^{(i)} \subset L^{(i)}$. Moreover we have $\hat{K}_{M/k} \supset \hat{K}_{L/k} = L$ by assumption. Hence $I_HA \subset B$. This implies $I_H^{i+1}A \subset I_H^iB = I_G^iB$, which means $K^{(i+1)} \supset L^{(i)}$.

(2) For $a \in A$, let $\tilde{a}$ be the class of A mod.I_HA which contains a. Let ϕ be a homomorphism of $H_0 \otimes (A/I_HA)$ to I_HA/I_H^2A defined by

$$\phi(\bar{\tau} \otimes \tilde{a}) = a^{\tau-1} \quad \text{mod.} I_H^2A$$

for $\tau \in H$ and $a \in A$. Then ϕ is well-defined and surjective by Furuta and Yamashita [2, Lemma 2]. Moreover [2, Theorem] implies

$$(2.1) \qquad \operatorname{Ker} \phi = \langle \prod(\bar{\tau} \otimes \tilde{a}_{\bar{\tau}}) \in H_0 \otimes A/I_HA \;; \prod a_{\bar{\tau}}^{\tau-1} = 1 \rangle .$$

Hence

$$G(K_{M/k}^{(2)}/\hat{K}_{M/k}) \simeq I_HA/I_H^2A \simeq \frac{H_0 \otimes (A/I_HA)}{\operatorname{Ker} \phi}$$

and

$$G(K_{M/k}^{(2)}/\hat{L}_{M/k}) \simeq I_HB/I_H^2A \simeq \frac{H_0 \otimes (B/I_HA)}{(\operatorname{Ker} \phi)\cap(H_0 \otimes (B/I_HA))} .$$

A canonical exact sequence $0 \longrightarrow B/I_HA \xrightarrow{i} A/I_HA \xrightarrow{\pi} A/B \longrightarrow 0$ implies an exact sequence

$$H_0 \otimes (B/I_HA) \xrightarrow{\mathrm{id} \otimes i} H_0 \otimes (A/I_HA) \xrightarrow{\mathrm{id} \otimes \pi} H_0 \otimes (A/B) \longrightarrow 0.$$

Put $\kappa = \mathrm{id} \otimes \pi$ and let ϕ' be the restriction of ϕ to $H_0 \otimes (B/I_HA)$. Then we have the following diagram of exact sequences

$$\begin{array}{ccccccccc}
 & & & & & & 0 & & \\
 & & & & & & \downarrow & & \\
0 & \longrightarrow & \mathrm{Ker}\ \phi' & \longrightarrow & H_0 \otimes (B/I_H A) & \xrightarrow{\phi'} & I_H B/I_H^2 A & \longrightarrow & 0 \\
 & & & & \downarrow i' & & \downarrow i'' & & \\
0 & \longrightarrow & \mathrm{Ker}\ \phi & \longrightarrow & H_0 \otimes (A/I_H A) & \xrightarrow{\phi} & I_H A/I_H^2 A & \longrightarrow & 0 \\
 & & & & \downarrow \kappa & & \downarrow j & & \\
 & & & & H_0 \otimes (A/B) & & I_H A/I_H B & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}$$

We define a homomorphism $\varphi_{A/B}$ of $H_0 \otimes (A/B)$ to $I_H A/I_H B$ by $\varphi_{A/B} \circ \kappa = j \circ \varphi$. Then $\varphi_{A/B}$ is well-defined and surjective. Moreover we have $\mathrm{Ker}\ \varphi_{A/B} = \kappa(\mathrm{Ker}(j \circ \phi)) = \kappa\phi^{-1}(\mathrm{Im}\ i'') = \kappa((\mathrm{Ker}\ \phi)(\mathrm{Im}\ i')) = \kappa(\mathrm{Ker}\ \phi)$. Therefor it follows from (2.1) and the definition of κ that $\mathrm{Ker}\ \varphi_{A/B} = R(H,A,B)$. Hence $G(\hat{L}_{M/k}/\hat{K}_{M/k}) \simeq I_H A/I_H B \simeq \dfrac{H_0 \otimes (A/B)}{\mathrm{Ker}\ \varphi_{A/B}} = \dfrac{H_0 \otimes (A/B)}{R(H,A,B)}$, which is to be proved.

Let $M \supset L \supset K \supset k$ be a tower of Galois extensions over k, and assume that $G(L/K)$ is contained in the center of $G(L/k)$. Let K' be the maximal abelian extension of k contained in K, and M' be the maximal abelian extension of K in M. For $\tau \in G(K/k)$ and $a \in G(M'/K)$, let $\bar{\tau}$ and $\bar{a}$ be elements of $G(K'/k)$ and $G(L/K)$ whose extensions are τ and a respectively. Set

(2.2) $$R(M,L,K,k) = \langle \prod(\bar{\tau} \otimes \bar{a}_{\tau}) \in G(K'/k) \otimes G(L/K);\ \prod a_{\tau}^{\tau-1} = 1,\ \tau \in G(K/k),\ a_{\tau} \in G(M'/K) \rangle.$$

Then we have

THEOREM 6. *Let* $M \supset L \supset K \supset k$ *be a tower of Galois extensions over* k. *Assume that* $G(L/K)$ *is cyclic and contained in the center of* $G(L/k)$. *Then*

$$G(\hat{L}_{M/k}/\hat{K}_{M/k}) \cong \frac{G(K'/k) \otimes G(L/K)}{R(M,L,K,k)} .$$

Proof. We apply Theorem 5 by setting $A = G(M'/K)$, $B = G(M'/L)$, $G = G(L/k)$ and $H = G(K/k)$. Then $R(M,L,K,k) = R(H,A,B)$ and $\hat{K}_{M/k} = \hat{K}_{M'/k}$ since $\hat{K}_{M/k}$ is abelian over K. Moreover $\hat{L}_{M/k}/K$ is abelian, for oviousely $G(\hat{L}_{M/k}/L)$ is contained in the center of $G(\hat{L}_{M/k}/K)$ and L/K is cyclic. Hence $\hat{L}_{M/k} = \hat{L}_{M'/k}$. Thus Theorem 5 implies the theorem.

Let $M \supset L \supset K \supset k_1 \supset k$ be a tower of Galois extensions over k, and assume that $G(L/K)$ is cyclic and contained in the center of $G(L/k)$. For $\tau_1 \in G(K/k_1)$, let $\bar{\tau}_1$ be the class of $G(K/k_1)$ mod. $[G(K/k_1),G(K/k_1)]$ which contains τ_1, and let $\bar{\bar{\tau}}_1$ be the class of $G(K/k)$ mod.$[G(K/k),G(K/k)]$ which containes τ_1. We define a homomorphism $\lambda_{k_1 \to k}$ of $R(M,L,K,k_1)$ to $R(M,L,K,k)$ by

$$\lambda_{k_1 \to k}(\bar{\tau}_1 \otimes \bar{a}) = \bar{\bar{\tau}}_1 \otimes \bar{a},$$

where $a \in G(M'/K) = G(M/K)/[G(M/K),G(M,K)]$ and $\bar{a}$ is an element of $G(L/K)$ whose extension to M' is equaql to a, M' being as above the maximal abelian extension of K in M. Then since L/K is cyclic, we have

$$(2.2') \qquad R(M,L,K,k) = \langle \bar{\tau} \otimes \bar{a} \in G(K'/k) \otimes G(L/K) \; ; \; a^{\tau-1} = 1, \; \tau \in G(K/k), \; a \in G(M'/K) \rangle.$$

This implies immediately

Theorem 7. *Let* $M \supset L \supset K \supset k$ *be a tower of Galois extensions over* k, *and assume that* $G(L/K)$ *is* cyclic and *contained in the center of* $G(L/k)$. *For* $\tau \in G(K/k)$, *denote by* K_τ *the intermediate field of* K/k *invariant by* τ. *Then we have*

$$R(M,L,K,k) = \overline{\prod_{\tau \in G(K/k)}} \lambda_{K_\tau \to k} R(M,L,K,K_\tau).$$

In the product, it is enough that τ *runs only over representatives of* $G(K/k)$ mod. $[G(K/k),G(K,k)]$.

§3. Reduction to extensions of type (ℓ,ℓ).

In order to prove Theorem 8, we prepare the following lemma.

LEMMA . *Let* K_1/k *be a cyclic extension of degree* ℓ. *Let* K_2/k *be a cyclic extension of degree a power of* ℓ, *and* F *be the extension* of k *of degree* ℓ *contained in* K_2 . *Put* $L = K_1K_2$, *and suppose* $K_1 \cap K_2 = k$. *If* M *is a Galois extension of* k *which contains* L *and abundant for* K_1F/k, *then* M *is also abundant for* L/k.

Proof. Put $L_1 = K_1F$. Let $\hat{L}_1$ and L_1^* be the maximal central and the genus field for L_1/k in M respectively. Since L_1/k is of type (ℓ,ℓ), the order of $H^{-3}(G(L_1/k),\mathbb{Z})$ is equal to ℓ. Hence $(\hat{L}_1 : L_1^*) = \ell$. Moreover L_1^* is the genus field for L/k in M, since the both L_1 and L are abelian over k. By the definition of $\hat{L}_1$, it is clear that $G(\hat{L}_1/L_1)$ and hence $G(\hat{L}_1/L)$ is contained in the center of $G(\hat{L}_1/k)$. Moreover $\hat{L}_1$ is non-abelian over k. Hence $\hat{L}_1$ is a non-abelian central extension for L/k. Since $H^{-3}(G(L/k),\mathbb{Z})$ is of order ℓ, the extension $\hat{L}_1$ must be the maximal central extension for L/k in M. This means that M is abundant for L/k.

THEOREM 8. *Let ℓ be a rational prime, and k_0 be an algebraic number field of finite degree satisfying* (*). *Then Leopoldt's conjecture is true for any algebraic number field* k *of finite degree which contains* k_0 *and* for ℓ *if and only if the following condition* (#) *is satisfied*:

(#) *Let* k *be any algebraic number field of finite degree which contains* k_0. *Let* L *be any abelian extension of* k *which is of type* (ℓ,ℓ) *and unramified outside* ℓ. *Then there always exists an abundant extension* M *for* L/k *such that* M *is also unramified outside* ℓ.

Proof. The necessity of (#) is trivial by Theorem 4.

Thus we prove the sufficiency. Let $k^{(\ell)}$ be as above the maximal ℓ-extension of k unramified outside ℓ, and K be a Galois extension of finite degree over k which is contained in $k^{(\ell)}$. To prove the theorem it is enough by Theorem 4 to show that $k^{(\ell)}$ is abundant for K/k. Since K/k is an ℓ-extension, there is a sequence $k = K_0 \subset K_1 \subset \cdots \subset K_t = K$ such that each K_i is normal over k, $(K_i : K_{i-1}) = \ell$, and $G(K_i/K_{i-1})$ is contained in the center of $G(K_i/k)$. Let $\bar{k}$ be the algebraic closure of k, and let R be as in (2.2).

We prove first the following equality

(2.3) $$R(k^{(\ell)}, K_i, K_{i-1}, k) = R(\bar{k}, K_i, K_{i-1}, k)$$

for $i = 2, 3, \cdots, t$.

For $\tau \in G(K_{i-1}/k)$, let K_τ be the intermediate field of K_{i-1}/k which is invariant by τ. Let M be any one of $k^{(\ell)}$ or $\bar{k}$. Then the maximal central extension $\hat{K}_{i-1,M/K_\tau}$ of K_{i-1}/K_τ in M is abelian over K_τ. Because $G(\hat{K}_{i-1,M/K_\tau}/K_{i-1})$ is contained in the center of $G(\hat{K}_{i-1,M/K_\tau}/K_\tau)$ and K_{i-1}/K_τ is cyclic. In the same manner, K_i is abelian over K_τ. Therefor $K^*_{i,M/K_\tau}$ and $\hat{K}_{i-1,M/K_\tau}$ are both the maximal abelian extension over K_τ contained in M. Hence $K^*_{i,M/K_\tau} = \hat{K}_{i-1,M/K_\tau}$. Then Theorem 6 implies

$$G(\hat{K}_{i,M/K_\tau}/K^*_{i,M/K_\tau}) = G(\hat{K}_{i,M/K_\tau}/\hat{K}_{i-1,M/K_\tau})$$

$$= \frac{G(K_{i-1}/K_\tau) \otimes G(K_i/K_{i-1})}{R(M,\ K_i,\ K_{i-1},\ K_\tau)}$$

On the other hand Theorem 7 implies

$$R(M,\ K_i,\ K_{i-1},\ k) = \prod_{\tau \in G(K_{i-1}/k)} \lambda_{K_\tau \to k} R(M,\ K_i,\ K_{i-1},\ K_\tau).$$

Therefor in order to prove (2.3) it is enough to show

$$(2.4) \qquad G(\hat{K}_{i,k^{(\ell)}/K_\tau} / K^*_{i,k^{(\ell)}/K_\tau}) \simeq G(\hat{K}_{i,\bar{k}/K_\tau} / K^*_{i,\bar{k}/K_\tau}).$$

Now if K_i is cyclic over K_τ, then $\hat{K}_{i,M/K_\tau} = K^*_{i,M/K_\tau}$ in both cases $M = k^{(\ell)}$ and $M = \bar{k}$. Hence (2.4) is trivial. Suppose that K_i is non-cyclic over K_τ. Then there is an intermediate field F_1 of K_i/K_τ such that $(F_1 : K_\tau) = \ell$, $K_i = F_1K_{i-1}$ and $F_1 \cap K_{i-1} = K_\tau$. Let F_2 be the intermediate field of K_{i-1}/K_τ which is of degree ℓ over K_τ. Then F_1F_2 is of type (ℓ,ℓ) over K_τ. Since $k^{(\ell)}$ is the maximal ℓ-extension over K_τ, $k^{(\ell)}$ is abundant for F_1F_2/K_τ by the assumtion (#) of the theorem. Similarly, $\bar{k}$ is also abundant for F_1F_2/K_τ by (1.1). Therefor it follows from Lemma that both $k^{(\ell)}$ and $\bar{k}$ are abundant for K_i/K_τ, which implies (2.4) and hence (2.3).

Next we prove that (2.3) implies Leopoldt's conjecture to be tru for k and ℓ. Now it follows from (2.3) and Theorem 6 that

$$G(\hat{K}_{i,k^{(\ell)}/k} / \hat{K}_{i-1,k^{(\ell)}/k}) \simeq G(\hat{K}_{i,\bar{k}/k} / \hat{K}_{i-1,\bar{k}/k})$$

for $i = 2, \cdots, t$. Hence we have

$$\prod_{i=2}^{t} (\hat{K}_{i,k^{(\ell)}/k} : \hat{K}_{i-1,k^{(\ell)}/k}) = \prod_{i=2}^{t} (\hat{K}_{i,\bar{k}/k} : \hat{K}_{i-1,\bar{k}/k}),$$

which implies

(2.5) $\quad (\hat{K}_{k^{(\ell)}/k} : \hat{K}_{1,k^{(\ell)}/k}) = (\hat{K}_{\bar{k}/k} : \hat{K}_{1,\bar{k}/k})$.

Let A_k and $A_k^{(\ell)}$ be the maximal abelian extension of k in $\bar{k}$ and in $k^{(\ell)}$ respectively. Then we have $\hat{K}_{1,k^{(\ell)}/k} = K^*_{1,k^{(\ell)}/k} = A_k^{(\ell)}$ and $\hat{K}_{1,\bar{k}/k} = K^*_{1,\bar{k}/k} = A_k$, for K_1 is cyclic over k. Hence (2.5) implies

(2.6) $\quad (\hat{K}_{k^{(\ell)}/k} : A_k^{(\ell)}) = (\hat{K}_{\bar{k}/k} : A_k)$.

Let K' be the maximal abelian extension over k contained in K. Then since $K^*_{k^{(\ell)}/k} = KA_k^{(\ell)}$ and $K_{\bar{k}/k} = KA_k$, we have $G(K^*_{k^{(\ell)}/k}/A_k^{(\ell)}) \simeq G(K/K')$ and $G(K^*_{\bar{k}/k}/A_k) \simeq G(K/K')$. Hence (2.6) implies

$$(\hat{K}_{k^{(\ell)}/k} : K^*_{k^{(\ell)}/k}) = (k_{\bar{k}/k} : K^*_{\bar{k}/k}).$$

The right hand side is equal to the order of $H^{-3}(G(K/k),\mathbb{Z})$ since $\bar{k}$ is abundant for K/k. This implies by Theorem 1 that $k^{(\ell)}$ is also abundant for K/k, which is to be proved.

Remark. When L/k is abelian of type (ℓ,ℓ), Schur's multiplicator $H^{-3}(G(L/k),\mathbb{Z})$ is cyclic of order ℓ. Therefor the existence of an abundant extension M for L/k in (#) is equivalent to the existence of a Galois extension $\hat{L}$ and an abelian extension L^* over k satisfying the following condition:

(i) $\quad \hat{L}$ is non-abelian over k, $\hat{L} \supset L^*$, and $(\hat{L} ; L^*) = \ell$,

(ii) $G(\hat{L}/L)$ is contained in the center of $G(\hat{L}/k)$,

(iii) $\hat{L}/L$ is unramified outside ℓ.

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